II

II Semester B.A./B.Sc. Examination, September 2020
(F + R) (CBCS) (Semester Scheme)
(2014-15 and Onwards)
MATHEMATICS (Paper – II)

Time : 3 Hours Max. Marks : 70

Instructions : *Answer **all** Parts.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Define subgroup of a group.

   b) In a group (G, $*$) $\forall$ a, b, c$\in$G prove that $a * b = a * c \Rightarrow b = c$.

   c) Find the radius of curvature at any point (p, r) on the curve $r^3 = a^2p$.

   d) Find the length of subtangent to the curve $r\theta = a$.

   e) Find $\frac{ds}{d\theta}$ if $r^2 = a^2\cos 2\theta$.

   f) Write the formula to find the volume of an arc of the curve $y = f(x)$ from $x = a$ and $x = b$.

   g) Find the integrating factor of $\frac{dy}{dx} + y\tan x = \sec x$.

   h) Solve $p^2 - 5p + 6 = 0$, where $p = \frac{dy}{dx}$.

PART – B

Answer **any one full** question : **(1×15=15)**

2. a) If (G, $*$) be a group and a, b$\in$G, then prove that $(a*b)^{-1} = b^{-1} * a^{-1}$.

   b) Prove that the set of all square roots of unity is a subgroup of the group of fourth roots of unity under multiplication.

   c) Prove that G = {2, 4, 6, 8} is a group under multiplication modulo 10.

OR

3. a) If G be the set of rationals except – 1 and $*$ is be the binary operation on G defined by $a*b = a + b + ab$, then prove that $(G, *)$ is a group.

   b) Prove that $G = \{1, 5, 7, 11\}$ is an abelian group under multiplication modulo 12.

   c) If $A = \{1, 2, 3\}$, $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ then find fof, gof and $(gof)^{-1}$.

PART – C

Answer **any two full** questions : (2×15=30)

4. a) With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$, for the polar curve $r = f(\theta)$.

   b) Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ cut orthogonally.

   c) Show that the radius of curvature at any point on the cardioid $r = a(1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.

OR

5. a) With usual notations, prove that the radius of curvature of the curve $y = f(x)$ is $\rho = \frac{\left(1+y_1^2\right)^{3/2}}{y_2}$.

   b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

   c) Obtain the pedal equation of the curve $r^2 = a^2 \cos 2\theta$.

6. a) Find all the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 - 3x - y - 1 = 0$.

   b) Find the surface area generated by revolving about the x-axis and the loop of the curve is $3ay^2 = x(x - a)^2$.

   c) Find the positions and nature of the double point of the curve, $x^3 + x^2 + y^2 - x - 4y + 3 = 0$.

OR

7. a) Find the length of an arc of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

   b) Find the envelope of the family of lines $y = mx + \frac{a}{m}$ where m is a parameter.

   c) Find the volume of the solid generated by revolving the curve astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

PART – D

Answer **any one full** question : **(1×15=15)**

8. a) Solve $\frac{dy}{dx}+\frac{1}{x}\sin 2y = x^3\cos^2 y$.

   b) Verify for exactness and solve $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$.

   c) Solve $y + px = p^2x^4$.

OR

9. a) Solve $\frac{dy}{dx}-\frac{2}{x}y=(x+x^2)$.

   b) Find the general and singular solution of $p^2(x^2 - a^2) - 2pxy + y^2 + a^2 = 0$.

   c) Prove that the family $y^2 = 4a(x + a)$ is self orthogonal.
